## SDU:

## Exercise 1:

Let $P_{1}$ and $P_{2}$ be two permutation matrices. Is $P_{1} \times P_{2}$ also a permutation matrix? Argue for or against your answer.

## Exercise 2:

1. Draw the graphs $G_{A}$ and $G_{B}$ for which the following 2 adjacency matrices $A$ and $B$ are given.

$$
A=\left(\begin{array}{llll}
0 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 \\
1 & 0 & 1 & 0
\end{array}\right) B=\left(\begin{array}{llll}
0 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 \\
0 & 1 & 1 & 0
\end{array}\right)
$$

2. Are the two graphs isomorphic?
3. How many different representations (in terms of adjacency matrices) of $G_{A}$ are there?
4. How many different representations (in terms of adjacency matrices) of $G_{B}$ are there?
5. Is there a permutation matrix $P$ such that $A=P(P B)^{T}$ holds?
6. If so, give all matrices $P$, such that $A=P(P B)^{T}$ holds.

## Exercise 3:

Given the following graph:


1. Give an adjacency matrix $A$ for the graph. (How many different are there?)
2. For your chosen adjacency matrix, how many permutation matrices $P$ are there, such that $A=P(P A)^{T}$ holds? (Remark: this number corresponds to the size of the so-called "automorphism group" of the graph).

## Exercise 4:

Given the following graph:


1. Give an adjacency matrix $A$ for the graph.
2. For your chosen adjacency matrix, how many permutation matrices $P$ are there, such that $A=P(P A)^{T}$ holds?

## Exercise 5*:

Given the following two graphs $G_{A}$ (left) and $G_{B}$ (right):


1. Give adjacency matrices for $G_{A}$ and $G_{B}$.
2. Is $G_{B}$ a subgraph of $G_{A}$ ?
3. How many different ways are there to find $G_{B}$ as a subgraph in $G_{A}$ ? (i.e., assuming as adjacency matrix $A$ and $B$ for graphs $G_{A}$ and $G_{B}$, how many leaf-nodes would the search the of the Ullmann algorithm have?)
4. How many different ways are there to find $G_{B}$ as an induced subgraph in $G_{A}$ ?

## Exercise 6:

The following is from the unit-testing of the graph theory assignment. Explain the expected result 10 .

```
>>> A = np.array([[ 0, 1, 0, 0, 1], \
    [ 1, 0, 1, 0, 0], \
    [0, 1, 0, 1, 0], \
    [0, 0, 1, 0, 1], \
    [1, 0, 0, 1, 0]])
>>> numIsomorphisms(A, A)
10
```


## Exercise 7*:

Use sigma aldrich https://www.sigmaaldrich.com/DK/en/structure-search to look for chemical structures. How many structures can you find which have the following as a substructure?


Can you find the price for the compound(s) you found? Do you know the compound with the highest similarity?

